

VU Research Portal

Teacher's epistemology and the monitoring of mathematical thinking in early years classrooms

van Oers, H.J.M.

published in

European Early Childhood Education Research Journal
2002

DOI (link to publisher)

[10.1080/13502930285208931](https://doi.org/10.1080/13502930285208931)

document version

Publisher's PDF, also known as Version of record

[Link to publication in VU Research Portal](#)

citation for published version (APA)

van Oers, H. J. M. (2002). Teacher's epistemology and the monitoring of mathematical thinking in early years classrooms. *European Early Childhood Education Research Journal*, 10(2), 19-30.
<https://doi.org/10.1080/13502930285208931>

General rights

Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

- Users may download and print one copy of any publication from the public portal for the purpose of private study or research.
- You may not further distribute the material or use it for any profit-making activity or commercial gain
- You may freely distribute the URL identifying the publication in the public portal ?

Take down policy

If you believe that this document breaches copyright please contact us providing details, and we will remove access to the work immediately and investigate your claim.

E-mail address:

vuresearchportal.ub@vu.nl

This article was downloaded by: [Vrije Universiteit Amsterdam]

On: 9 June 2011

Access details: Access Details: [subscription number 932819835]

Publisher Routledge

Informa Ltd Registered in England and Wales Registered Number: 1072954 Registered office: Mortimer House, 37-41 Mortimer Street, London W1T 3JH, UK



European Early Childhood Education Research Journal

Publication details, including instructions for authors and subscription information:

<http://www.informaworld.com/smpp/title~content=t776628938>

Teachers' epistemology and the monitoring of mathematical thinking in early years classrooms

Bert van Oers^a

^a Free University Amsterdam, The Netherlands

To cite this Article van Oers, Bert(2002) 'Teachers' epistemology and the monitoring of mathematical thinking in early years classrooms', European Early Childhood Education Research Journal, 10: 2, 19 — 30

To link to this Article: DOI: 10.1080/13502930285208931

URL: <http://dx.doi.org/10.1080/13502930285208931>

PLEASE SCROLL DOWN FOR ARTICLE

Full terms and conditions of use: <http://www.informaworld.com/terms-and-conditions-of-access.pdf>

This article may be used for research, teaching and private study purposes. Any substantial or systematic reproduction, re-distribution, re-selling, loan or sub-licensing, systematic supply or distribution in any form to anyone is expressly forbidden.

The publisher does not give any warranty express or implied or make any representation that the contents will be complete or accurate or up to date. The accuracy of any instructions, formulae and drug doses should be independently verified with primary sources. The publisher shall not be liable for any loss, actions, claims, proceedings, demand or costs or damages whatsoever or howsoever caused arising directly or indirectly in connection with or arising out of the use of this material.

Teachers' Epistemology and the Monitoring of Mathematical Thinking in Early Years Classrooms

BERT VAN OERS

Free University Amsterdam
The Netherlands

SUMMARY: This article presents an argument that holds that epistemology is involved in different ways in discussions on education. Taking mathematics education as an object of study, it will be argued that the teacher's continuous monitoring of pupils' mathematical activity is guided by a view on mathematics (the teacher's mathematical epistemology). This will be illustrated on the basis of assessments of early mathematical development in a play-based curriculum. On the basis of reports by teachers on the mathematical development of their pupils (collected over three years in early year's classrooms in a Dutch school) it can be demonstrated that different epistemologies (a content oriented epistemology and an activity oriented epistemology) are involved. It seems plausible to assume on the basis of our data that further advancing the teachers' mathematical epistemology may contribute to the quality of the teachers' continuous monitoring and guidance of mathematical thinking of pupils.

RÉSUMÉ: Cet article argumente l'idée selon laquelle l'épistémologie peut être impliquée de différentes façons dans les discours sur l'éducation. En prenant le cas des mathématiques, il est avancé que l'organisation des activités mathématiques par l'enseignant se fonde sur sa conception des mathématiques (l'épistémologie mathématique de l'enseignant). Cette hypothèse est illustrée à l'aide de rapports d'évaluation d'activités mathématiques de jeunes enfants, dans le cadre d'un programme basé sur le jeu. À partir de ces données, recueillies pendant trois ans dans les premières classes d'une école néerlandaise, il est possible de montrer la présence de différentes épistémologies mathématiques (une épistémologie orientée sur le contenu et une épistémologie orientée sur les activités). On peut penser que les avancées de l'épistémologie mathématique des enseignants peuvent contribuer à la qualité de leur accompagnement de la pensée mathématique des élèves.

ZUSAMMENFASSUNG: Es wird argumentiert, dass Epistemologie auf unterschiedliche Weise in Diskussionen über Bildung und Lernen eingeht. Am Beispiel Mathematikunterricht wird ausgeführt, dass die kontinuierliche Beobachtung und Überwachung der mathematischen Aktivitäten der Schüler durch die Lehrer durch deren Mathematik-Epistemologie geleitet wird. Dies wird anhand von Messungen der frühen mathematischen Entwicklung in einem spielorientierten Curriculum illustriert. Berichte der Lehrer über die mathematische Entwicklung ihrer Schüler, die über drei Jahre hinweg in einer niederländischen Schule gesammelt wurden, kann gezeigt werden, dass unterschiedliche Epistemologien – eine inhaltsorientierte und eine aktivitätszentrierte Epistemologie – beteiligt sind. Auf der gegebenen Datenbasis erscheint plausibel, dass eine Förderung der mathematischen Epistemologie der Lehrer sich positiv auswirkt auf die Qualität ihrer Beobachtung und Anleitung des mathematischen Denkens der Schüler.

RESUMEN: En este artículo se argumenta que la epistemología está presente de distintas maneras en las discusiones sobre la enseñanza. Por ejemplo, resulta plausible que las observaciones del profesorado sobre las actividades matemáticas del alumnado están orientadas por su visión de las matemáticas (la epistemología matemática del profesorado). En este artículo se demuestra dicha hipótesis a partir de las valoraciones llevadas a cabo por profesores sobre las actividades matemáticas de alumnos en un currículo de orientación lúdica. A partir de los informes del profesorado acerca del desarrollo del pensamiento matemático del alumnado (recopilados durante tres años en el primer ciclo de una escuela de enseñanza primaria holandesa), podemos demostrar distintas epistemologías (una orientada hacia el contenido y otra orientada hacia las actividades). Basándonos en los datos de nuestro estudio podemos concluir que el apoyo al desarrollo de la epistemología matemática del profesorado puede tal vez contribuir a la mejora de las observaciones de dicho profesorado, lo cual podría ejercer asimismo un efecto positivo sobre la validez del currículo de orientación lúdica para el estímulo del pensamiento matemático en criaturas de corta edad.

Keywords: Assessment; Epistemology; Teacher; Vygotsky; Mathematics.

1. Concerns about a play-based curriculum

One of the approaches to early years education in the Dutch primary school (grades 1 - 4) is based on an educational view that draws its main concepts from Vygotskian theory and its later sociocultural developments. This approach is now generally called 'Developmental Education' (Ontwikkelingsgericht Onderwijs), and it has been elaborated in the Netherlands into a specific curriculum strategy called 'Basisontwikkeling' for the lower grades of primary school which is currently being implemented in a growing number of schools (see Janssen-Vos, 1997). An elaboration of the developmental education concept for the upper grades is currently under way.

One of the general characteristics of this curriculum approach is its commitment to the belief of the developmentability of children: development can be promoted by getting children involved in sociocultural activities in which they are allowed to participate in a playful way, and in which the teacher is involved as one of the more knowledgeable participants. The playful participation of the children in these sociocultural activities is assumed to result in a zone of proximal development in which the children can appropriate (with the help of the teacher and peers) in a meaningful way the cultural means (tools) for reading, writing, mathematising, etc. The curriculum strategy is intended to produce a *play-based curriculum* that organises itself in the interaction between teacher and pupils. An essential aspect of a play-based curriculum is that it allows pupils a certain degree of freedom in the accomplishment of their activity. It is the teacher's task to watch over the cultural relevance of the course of learning (heading to culturally relevant goals), and to sensitively take into account the pupils' interests and abilities (see Van Oers, 1999a).

It stands to reason that this lays a heavy burden on the teachers' shoulders. A lot of energy has been invested over the past ten years to assist teachers in their daily practices with the implementation of this play-based curriculum and its educational philosophy. Concrete supervision and team guidance have been organised by several educational institutions, conferences for teachers have been organised by a Dutch institution for the innovation of education (APS), and by the Association for Developmental Education (Academie voor Ontwikkelingsgericht Onderwijs), books and articles have been written for the explanation of the basic principles, and for the illustration of these principles in examples for good practice (Knijpstra, Pomper & Schiferli, 1998; Fijma & Vink, 1998).

Of course, the question of the quality of this educational approach has been asked frequently and more and more urgently nowadays. Like most Western cultures, the Dutch society strongly favours accountability for its public institutions. With regard to the play-based curriculum 'Basisontwikkeling' this resulted, for example, in questions about the learning outcomes for children with different abilities and cultural backgrounds. Although this is certainly a justified and very important question, answering these questions raises serious problems with regard to the

assessment of the learning outcomes. There is considerable evidence that the style of assessment has a significant effect on the learning outcomes: the type of performances that the students have to accomplish during the test tend to become an important determinant for the type of activities that students have to complete during the lessons, and for the type of values that they are expected to adopt (see for example Clarke, 1996). So there are good reasons to reflect seriously on the correspondence between the curriculum and its testing procedures.

Due to the acknowledgement of the intimate relationships between the assessment procedures and the learning processes in the curriculum, the nature and the qualities of the assessment procedures became more and more a serious concern in the recent developments of the play-based curriculum. The present article deals with an aspect of this question and analyses the teachers' notions about mathematising, since these notions are basic tools in the teachers' observations of children's mathematical development. So, my focus will be mainly on the **validity** of the teachers' observational strategies, considering the play-based curriculum. Therefore I will address the question of what is required of a teacher who wants to assess pupils' mathematical development in the play-based curriculum? Before coming to this question a brief explanation of the theoretical background of the study is necessary (Section 2).

2. Assessment in a play-based curriculum.

Many teachers and developers of the play-based curriculum consider the use of standardised tests as inappropriate. A standardised test confronts children with test situations that do not make any human sense to them as the test items are not functionally related to an activity they are intrinsically interested in, nor to the format they are familiar with. Hence, people are concerned that the outcomes from these tests will not give a fair (often probably an underestimated) estimation of the pupils' abilities and developmental process. The curriculum '*Basisontwikkeling*' itself provides an observation strategy that assists teachers in observing children in their daily play activities with regard to a number of points that are considered to be important indicators of the child's development. This observation strategy is called HOREB, which is a Dutch acronym for 'Action Oriented Observation, Registration, and Evaluation in Basisontwikkeling' (Janssen-Vos, Pomper, & Schiferli, 1998). For practical reasons I shall use this name HOREB to refer to the assessment strategy connected to this play-based curriculum. I cannot, however, make an attempt here to explain the details and rationales of this observational strategy. In general it may be said that the strategy covers broad developmental aspects (such as the abilities to participate in different activities like communicating, problem solving, reflecting, creative constructing etc.), as well as subject matter-bound specific abilities and knowledge (see Janssen-Vos, 1997). Importantly, the HOREB observation strategy is not only oriented at subject-matter operations, but also tries to register general developmental qualities (including interests and attitudes).

It is important to note here, that the proposed manner of observing children in their daily activities is not a passive registering by the teacher of what she happens to see. It involves an active exploratory way of observing children, probing new actions or cultural tools with the children that seem to be functional in the activities at hand, and noting how the children respond to these measures. As an example one could imagine a classroom project of establishing a museum in the school by the children from grade 2 and 3. Of course, at some moment during the project the need arises of making program books for the museum, invitations, labels for the objects etc. This introduces in a 'natural way' an element of writing in the children's activities. With regard to these writing actions the teacher sets new criteria or introduces new tools or rules in order to observe how children deal with these new elements, where they need help (if any), and how much. As to the invitations, for instance, for other children to visit the museum the teacher requires that a child indicate as exactly as possible the time when the museum is open. By so doing she can observe how a child picks up this assignment, and what kind of help she needs. Maybe she decides that the child is not yet able to deal with this kind of knowledge, and she decides to come back to it later in another activity in a bit less complicated way.

This kind of ‘participatory testing’ by the teacher(s) is based on direct and continuous monitoring of the children in their play activities. Experiences thus far have demonstrated that teachers can develop impressive abilities in this participatory testing and observing children. However, there is still a public concern (in parents, teachers, directors, inspectorate) with regard to the qualities of the resulting learning processes, and to the reliability and validity of the assessment procedures used. Recent evaluative studies, however, could show that pupils from a play-based curriculum (grades 2 - 4, ages 5 to 8) on average could learn to read and mathematise as well as pupils from a more traditional curriculum, measured by standardised tests. Moreover, it turned out that the teachers’ observational strategy is equally able to identify the good, or the ‘at risk’ children with regard to reading (see van Oers, 1999b), as the standardised tests do. Hence there is reason to assume that the outcome level of the play-based curriculum is acceptable, as well as the reliability of the observational assessment procedure.

Seen from a conceptual point of view, there is, however, still the question of the validity of the teachers’ observations: do they indeed observe the qualities that are to be considered essential for a developmental description from a Vygotskian point of view? Or are they merely employing an interiorised version of the publicly available standardised tests (which might explain the correspondence between the two measures)?

In this article I want to report some initial results of a study of this validity question with regard to the mathematical development of pupils in grades 2 to 4 in a Dutch primary school committed to this play-based curriculum already for several years.

3. Monitoring mathematical development

Epistemological and psychological studies have argued that observing is not a value-free process. People register those elements from their environment that first of all have significance for themselves. Consequently, what a person reports having observed is a function of the idea of that person concerning what is relevant and irrelevant for the object at hand and for the aims of the observations. It is clear then, that a teacher, when observing a child to estimate its level of mathematical development, reports those aspects of the child’s activity that he or she thinks is relevant for the understanding of the child’s **mathematical** ability. Hence, an idea of what mathematics is about is always involved. A view about mathematics, what it is, what it should be, what it entails, how it is to be carried out, is called here a *mathematical epistemology*. My assumption is that a teacher’s own (implicit or explicit) mathematical epistemology is an essential tool for the assessment of children’s mathematical ability by a participatory observation and testing strategy.

But, of course, it is not only the teacher’s private epistemology that is involved here. This mathematical epistemology is itself a learning result that the teacher has gained during his or her own schooling and professional development. It shouldn’t come as a surprise that most teachers’ mathematical epistemology is akin to the dominant vision on mathematics as it is propagated in many teacher-training institutes. Perhaps, it even carries the remnants of the image of mathematics as it was embodied in the mainstream methods for mathematics teaching in their own primary school days. To begin with, the teacher’s mathematical epistemology is –what we could call in Bakhtinian terms- multivoiced or polyphonic. The voices of his or her masters, colleagues, supervisors, favourite curriculum developers etc probably echo in the teacher’s mathematical epistemology. And these voices even may not always be consonant. Epistemologies may contain contradictory elements. For example, when teachers say (or act as if) knowing the Basics by heart is the most important element of beginning mathematics, this may contradict the conviction that insightful problem solving is an essential element of mathematical development.

When learning to use the HOREB strategy, the teachers are also confronted with a new mathematical epistemology that is embodied in this strategy and that is supposed to be consistent with the (Vygotskian) activity based educational philosophy (“Developmental Education”) behind the curriculum. The Developmental Education view endorses a view on mathematics that is activity based, and that comes down to a way of solving quantitative and spatial problems that are entailed in a meaningful situation, with the help of schematic tools (symbolic models) and collec-

tive reflection on those tools for further improvement and schematisation. These characteristics make the Vygotskian approach very easily reconcilable with another activity-based approach that has become very popular and influential in the Netherlands: the *Realistic Mathematics Education* (Gravemeijer, 1994; see also Cobb, Gravemeijer, Yackel, McKlain, & Whitenack, 1997). Essential for this mathematical epistemology is that mathematics is a cultural activity that should not be reduced to correctly performing mathematical operations. Besides an emphasis on activity, problem solving, reflection, discourse, symbolising and modelling, the epistemology also acknowledges the broader sociocultural context of mathematising and is sensitive to sociomathematical norms and personal orientations in the problem solving process.

On the basis of such *activity oriented mathematical epistemology*, the HOREB strategy includes suggestions for the teacher to make observations in the context of the child's play on most of the above-mentioned points. Of course there is attention to operational aspects of counting, but also to the more 'dynamical' aspects of mathematical activity, such as making schematic representations, establishing abstract relations; finally, attention is also paid to attitudinal aspects expressed in interest in questions of certainty (cf. also van Oers, 1994; 1996). See Figure 1 for a more detailed overview of the points suggested by the HOREB manual for the observation of pupils' mathematical development.

Horeb's Observational Foci

A. Meaning and motives of children

Possible points for attention that may signal development:

- manifestations of mathematical actions in the children's play (number use, measuring, etc);
- interest for mathematical activities, and tools, strategies;
- a need for 'knowing sure', for inquiry, reflection with the help of mathematical language and tools;
- mathematical activities themselves are motive and context for action.

B. Development of mathematical activities

Possible points for attention that may signal development:

- increasing need for knowing sure, signalled in inquiry, comparison, reflection, controlling (from horizontally mathematising, to vertically mathematising and recontextualisation);
- improving mastery of symbols and symbolic representations:
 - from personally invented representations to conventional mathematical notations;
 - from making representations to interpreting symbolic models;
 - from representing static representations to dynamic representations.
- improvements in the use of theoretical relations:
 - from incidental use to deliberate use;
 - from guessing to prediction.

C. Language and thinking in mathematical activities

Points for attention that may signal development:

- increasing use of mathematical language (for operations, relations, concepts, rules);
- improving quality of analysing, strategically approaching problems, solving problems, formulating hypotheses;
- negotiation of meanings;
- increasing use of symbols, representations, thought experiments;
- increasing reflection as a means for self-regulation and control.

D. Broad developmental aspects

Points for attention that may signal development:

- communication and language become more mathematical for the solution of problems;
- increasing self-control and regulation;
- reasoning and arguing (as means for 'knowing for sure');
- collaboration and negotiation of meaning.

E. Specific mathematical knowledge and abilities

Points for attention that may signal development:

- numbers and operations:
 - knowledge of numerals (to 20), adding and subtracting (-20);
 - knowledge of number line (-100), adding and subtracting (-100);
 - understanding of multiplications, tables.
- measuring & geometry:
 - use/invention of units of measurement in different areas;
 - use of tables and graphs;
 - interpretation of tables and graphs/symbolic representations.

FIGURE 1: Possible points of attention for observation of mathematical activity according to the HOREB strategy

Obviously, when having to appropriate the HOREB strategy for observing mathematical development, the teachers need to incorporate the realistic mathematics epistemology into their existing mathematical epistemology. It is plausible to assume that this is a gradual process going from mixed epistemologies in the beginning towards a more coherent mathematical epistemology that is more and more in accordance with the activity oriented epistemology favoured by the play-based curriculum approach.

Surely, other mathematical epistemologies will be encountered by the teachers, e.g. when they apply a standardised test in their classroom practice. The standardised tests that were used in our research as an object of comparison were developed by the Dutch National Institute for Test Development (CITO). These tests focus on mathematical thinking of pupils that is expressed in their abilities to perform (pre) mathematical operations correctly: they focus on standard mathematical prerequisites (classification, seriation, knowledge of numerals, etc), or on standard arithmetical operations (counting, adding, subtracting, multiplication). No attempt is made in these tests to examine directly the problem solving process of the pupils, nor their mathematical interests or attitudes. The belief that such tests will give a reliable view on mathematical development is, according to the developmental education view of mathematical development, limited and an example of a quite different ('foreign') mathematical epistemology. An epistemology that can justifiably be called a *content oriented mathematical epistemology*. It is obvious, then, that the administration of such tests in a developmental education context will cause contradictions in the curriculum (both for the pupils and the teacher), and the test results will not be very easy to accept as a description of a pupil's mathematical development. The tension between the different mathematical epistemologies may be another reason that contributes to some teachers' reluctance to use the standardised tests as a tool for diagnosing the pupils' development.

In the next section I will report some data concerning the development of teachers' mathematical epistemology in relation to their continuous monitoring of pupils' mathematical activity in the context of play. Hence the focus will be on the conceptual validity of the teachers' observations in the context of this curriculum strategy. The comparison of the developmental estimates from the two epistemological backgrounds will be reported elsewhere.

4. How teachers monitor pupils' mathematical development

In our research project we asked teachers to write summarising reports of their ongoing observations of pupils in the daily classroom activities, independent from the standardised tests that were also administered. We made an agreement with the teachers that the results of the standardised tests were not communicated to them in order to prevent as much as possible a contamination of their own evaluations by these outcomes. Over a period of three years (1997 - 2000) the tests (reading and math) were administered in grades 2, 3 and 4 (ages 5 - 8). The writing of reports was organised in shifts: In the first year the teachers (N=2) from grade 2 wrote developmental reports for the research; in the second year the teachers from grade 2 and 3 wrote developmental reports; in the final year the teachers from grade 2, 3 and 4 (N= 7) were involved in report writing. The teachers from the first year were still involved in the last year of the investigation, which gives us the opportunity to follow their development over the years. At the end of every year the teachers got general feedback about their reports. In this meeting all the teachers involved participated as well as the investigator, and a teacher trainer from a teacher-training institute who supervised the school's development. Several problems and good examples of observation of pupils in their daily activities were discussed during the meeting. These meetings were aimed at giving teachers feedback and endorsement for their efforts. They also had an instructive intention aiming at the further improvement of the teachers' observational strategies, according to the educational and subject matter concepts behind the HOREB instrument. We found it important that the teachers were not just mechanically filling in checklists, but could use the manual as a meaningful tool for their own observational activities.

The reports of the teachers were written as brief texts summarizing developmental events that the teacher found significant over the past few months. The texts were written in such a way that they would be readable for the investigator (no short-hand or just key words). Taking one of the last rounds of reporting as an example, the following overview might give an impression of the formal characteristics of the texts:

TABLE 1: Characteristics of teacher reports

Teacher	Number of reports (in words)	average length (in number of words)	dispersion
A *(grade 2)	11	43	25 - 64
A* (grade 3)	12	64	45 - 101
B* (grade 2)	3	137	119 - 163
C (grade 2)	6	44	41 - 56
C (grade 3)	5	40	33 - 42
D (grade 2)	5	36	28 - 50
D (grade 3)	6	45	34 - 70
E (grade 2)	3	61	50 - 68
F (grade 4)	17	87	47 - 121
G (grade 4)	18	40	17 - 90

Remarks:

- Most of the time more than one teacher is taking care of a class (part time); most of the grades are part of a combined class (combining grade 1 and 2; grade 2 and 3; grade 3 and 4; grade 4 and 5;
- The teachers with * were involved from the beginning of the investigation (the so-called pioneer teachers);

- Average length of texts (overall): 62 words
- Number of pupils per grade: grade 2: 28; grade 3: 23; grade 4: 35.

The reports were analysed in a number of different ways. In this article I will only report some data based on the teachers' reports that give us some insight into the teachers' mathematical epistemology. To begin with, it is remarkable that the teachers in the beginning always tend to give extensive reports on general developmental features of the pupils and on literacy, while far less information was given on mathematical development. As to mathematical development of the 5-year-old pupils (grade 2) the teachers at first generally restricted themselves to:

- remarks about knowledge of numerals;
- arithmetical vocabulary (more, less, bigger than etc);
- ability of counting (forward and backward);
- level of involvement of pupils in counting activities ('interest').

The bias of the teachers' reports towards literacy development is explicable on the basis of the history of the project: a lot more attention was given to the language part of the children's development. This might be due to the strong language orientation of the Vygotskian approach. The inclusion of information about the pupils' involvement in activities is clearly also a consequence of the general educational philosophy. On the assumption (see above) that people tend to register those things that they think are relevant for their goals, we may say that for the rest the teachers' descriptions of the mathematical development showed a traditional content and operation oriented epistemology. It is remarkable that the teachers even made little use of the observational suggestions given in the HOREB manual. Probably it is not too wild a guess that at that time the issues in the manual (about e.g. relations, representations, 'are-you-sure?-question) did not yet fit very well into their personal mathematical epistemology. As a result, the teachers probably could not use these effectively at that time for their own observations in the classroom.

In the feedback about these reports, we decided to work with the teachers in a session on the HOREB manual, explaining and illustrating the elements in the manual in terms of possible observations of mathematical activities that they could have made during their projects. In this process the teachers came with questions of how to interpret activities of children, how to evoke new actions (and which actions?) in order to 'test' the pupils willingness to get engaged in new mathematical actions. It is evident that the teachers were trying to elaborate their view on mathematics. They got convinced that mathematical activity for these young kids was more than just being able to count or read and write numerals.

The results of these developmental processes can be seen in the teachers' reports at the end of the project, which contrast remarkably with the reports from the beginning. Typical examples of two teachers (involved from the beginning of the experiment) in the third year (February 2000) were the following:

About one of her grade 2 pupils (5; 9) one of the teachers writes the following report (Febr. 2000), going evidently far beyond just mentioning effectiveness: in computation,

"Danielle has a good understanding of numbers. She can count to 100 and backwards. She understands where to put the numbers on the number line. She can fill in a table without help, and she understands what you can read from a graph. She completed all symmetry problems in the workbook on her own"

About a grade 3 pupil (6;7) she writes (Febr. 2000):

"Lois likes to make mathematical tasks. She has a good understanding and can fill in a graph independently. She can put co-ordinates of a map in a table. She loves it to invent new tasks with her friends and then work them out into a table or a map. No trouble with symmetry problems. Her workbook on clocks looked fine. She has a good sense of time"

Or:

“Katja calculates easily and fast in her head. She understands the numbers until 100 well. She has a good understanding of numbers. She can even count forward in steps of three and four. She understands the meaning of a graph, and has no problems with tasks on symmetry and co-ordinates”

This latter description illustrates another interesting point. It is to be expected that the teachers themselves also develop in their way of writing about children. It goes without saying that the teachers cannot always describe all things that they have observed. In the third grade the teacher doesn't report that a pupil knows the names of the numerals and can write them. Reference to these abilities is only made, when a pupil still can **not** read or write the numerals. The mastery of these elementary abilities is taken for granted when the pupils can accomplish other more difficult tasks and the teacher does **not** mention them. Another interesting way of implicitly referring to fundamental operations is illustrated above in the report about Katja: the remark “*She has a good understanding of numbers*” is actually a summarising way of saying that she masters the elementary operations regarding number. And from the previous reports of this teacher (concerning reports of the grade 2 pupils) we know that this means that the pupil can count fluently (back and forth), can link numerals with the corresponding quantities, can invent ways of representing numbers, can put numbers on a number line, can split quantities etc. The teacher thus summarises elementary developments in a comprehensive way. The development of the teachers' mathematical epistemology also includes the construction of such ways of talking about mathematics in a comprehensive way. And it is clear for this teacher, this implies more than just being able to count or make the assigned computational tasks.

An analogical development we can see in the other teachers as to their mathematical epistemology: beginning with a view on mathematics very close to a traditional, operation/content oriented view, we see that over time the epistemology broadens towards a conception of mathematics that gives more attention to systematic problem solving with the help of symbolic representations that inscript quantitative or spatial relations. As a result, the teachers report more dynamical aspects of mathematical activities in addition to the mastery of arithmetical operations.

The ‘pioneer’ teachers who practised this writing of developmental reports over the past three years had a very important role to play in the development of the team's mathematical epistemology and the other teachers' style of observing and registering developments. It is clear that other beginning teachers could make a head start when they started writing about pupils, due to the help that they got from these ‘pioneer’ teachers. These newcomer teachers started with a broader conception of mathematics, and tried to observe more than just the computational fluency of the pupils: they show attention to representational activities for example, or for attitudinal aspects in mathematical activity (“he wants to be sure of the correctness of his solutions”). Nevertheless, their reports are sometimes also a bit ambivalent in the beginning. A typical example is the following report of a beginning teacher on a grade four child (7;5):

“Lieke doesn't work very fast, but she doesn't make many mistakes. She can recognise the hours, halves and quarters on the clock; she doesn't like to make sums, she has problems with multiplication”

Or (from a report on another fourth grade child – 7;10) by the same teacher:

“It takes long before she understands a new topic. She needs a lot of help. She uses material support. She is unable to recognise written numbers (higher than 10) without material support. We do a lot of counting with her. Her development goes on, but slowly. The construction work is still very much in the two-dimensional plane [...]; she does not make symbolic representations of these constructions”.

Another grade four teacher developed a good sense for representational activities (mapping, graphs etc) and she reports often about these activities of the children. However, her way of reporting

about the mathematical development of the pupils suggests that she still draws a strict demarcation line between plain arithmetic and mathematics (including the problem solving and representational activities). Typically she writes:

“Wai-Ho listens concentratedly to the arithmetical instructions. In general he understands them well. He asks for additional sums for practising, for he loves doing arithmetic. For mathematical activities he shows less interest”

Many of the teachers in grade four report about the mastery of operational abilities of the children (and this is understandable given the broader institutional system they are also part of). They seem to make a distinction between arithmetic and mathematics in a more broad sense. In the first year of their report writing (2000) they never reported on the abilities of the children with regard to the dynamical aspects of number use (representing number operations, model making, problem solving strategies, mathematical attitude). Presumably they still have a mixed (*‘polyphonic’*) mathematical epistemology, bringing together an operation/content-oriented view on mathematics with a more dynamic (realistic view). In their reports these teachers focus mainly on the operational aspects of arithmetic and add some elements about representational activities and symbolising. Their continuous monitoring of the children’s activities in the classroom is presumably also biased towards getting information about the basic operations, without attempts to link these operations for the children to the more dynamical aspects of mathematical thinking, nor exploring the mathematical aspects of their computations. Their reports do suggest that this is indeed the case.

An important, general and outstanding finding for all teachers is the **stability of their way of analysing**: a teacher who reports ‘counting backwards’, or ‘reading tables and graphs’, or some other characteristic as a hallmark of mathematical thinking development, tends to use these criteria in all her observations of children’s activities. This supports the idea, that the developmental reports indicate a more or less stabilised *observational style* of the teacher. This is also consistent with the idea of a mathematical epistemology as a basis for observation.

5. Conclusion and discussion

Our theoretical expectation of different mathematical epistemologies as one of the tools for a teacher’s continuous monitoring of mathematical development of children could be corroborated by an analysis of teachers’ reports on the mathematical development of pupils. On the assumption that teachers register mainly those elements of a child’s activity that they believe are relevant for mathematical development, it can be argued by implication that teachers producing substantially different reports also hold different mathematical epistemologies. Although this style of reporting seems to be rather stable for a teacher over the different reports and even over different moments in the school year, epistemologies are open for development themselves. At first this may lead to ‘polyphonic’ epistemologies expressing different views on mathematics at a time. A further integration of the different mathematical voices is probably a process that takes more time, discourse and guidance. The social space of the team’s culture is presumably very important for the integration of the different ‘texts’ about mathematics, as well as for learning to deal effectively with ‘the unavoidable tension’ between tools (here: epistemologies) and classroom action (cf Wertsch, 1998).

The attempt to understand teacher’s activities in terms of teacher epistemologies is gaining growing attention over the last decade, both in general (see Hofer & Pintricht, 1997), and in relation to mathematics education (see for example: Fennema & Loef, 1992; Thompson, 1992). Interestingly, Verschaffel, Greer, & De Corte (2000) recently produced supporting evidence for the impact that a teacher’s particular mathematical epistemology may have on conceptions that pupils develop about mathematical problems. Hence, the pieces of evidence that suggest that the teacher’s epistemology has a determining influence on pupils’ thinking are accumulating.

In another article (van Oers, 1999b) I reported an investigation (with the same group of teachers) on the observation of literacy development in pupils by teachers, that may contribute to this very same issue. In general a similar development could be found in this literacy-investiga-

tion: over time teachers tend to report more substantial categories of literacy, demonstrating that their view on literacy develops away from a mere phonics view, towards a more communicative and semiotic view, which is more consistent with the view that is embodied in the play-based curriculum. The present findings strongly support this very same supposition that teachers by team interaction and guidance develop their idea (of literacy and mathematics) more in accordance with the views that are espoused by the curriculum.

In a general way, our findings also seem to converge with the findings of Fuchs, Fuchs, Karns, Hamlett, & Katzaroff (1999). These researchers investigated the effects of what they call Performance Assessment in the mathematics curriculum in elementary school. Performance assessment is a way of assessing the learning outcomes of pupils in a curriculum with the help of authentic problems in which the students are supposed to develop solutions involving the application of multiple skills and strategies (Fuchs et al., 1999, p. 611). However, there is also a lot of evidence for the fact that merely making tasks more 'authentic' will by itself not dramatically change the students' performances. This was for example one of the results from a study of Verschaffel et al. (2000). These authors furthermore provided evidence that it is a change in a student's belief systems concerning mathematics that is needed. Performance assessments - as applied by Fuchs et al - by themselves will not change the students belief system, as the problems of this assessment still turn out to be quite content oriented, and as such close to a traditional, school-like mathematical epistemology. One could assume that the teachers in Fuchs' study did not have to struggle with attaining a new epistemology. Broadening their traditional operation-oriented scope on mathematics probably would be enough.

According to Fuchs et al, performance assessments have to be administered repeatedly during the curriculum so that the teacher can redirect his/her curriculum plans, more in accordance with the needs of the students and the aims of the (reform) curriculum of mathematics education. One of the findings of the Fuchs et al. study was that teachers as a result of Performance Assessments indeed tend to change their curriculum plans from less emphasis on number facts, basic skills, routine computation towards an increasing focus on mathematical communication and problem solving. However, as was pointed out above, we may assume that the effect of this will still be limited as long as the teachers do not adjust their mathematical epistemology. On this point, our and Fuchs' approaches show their differences. In our play-based curriculum we tried to assist teachers in appropriating a new mathematical epistemology. Moreover, the nature of the problems used for the 'participatory testing' was closer to the activities of the children themselves, which results in more authentic activities and developmentally appropriate testing. The problems in our classrooms probably made more sense to the pupils than the teacher-chosen problems for overall classroom testing in the Fuchs' et al. study.

Unfortunately Fuchs et al. do not try to interpret their findings in terms of teacher epistemologies. Although we have to be very cautious with sweeping conclusions regarding the teachers' epistemologies, it is still striking that teachers report with such consistency particular things about pupils. A next step in confirming our assumption about the teachers' epistemologies is gathering information about the teacher's ideas on mathematics via different approaches (interview, classroom observation). Although we cannot be absolutely sure at this moment that different mathematical epistemologies in teachers may (partly) explain their different styles of observing and reporting, it is remarkable that the teachers pay attention to different actions and that their styles become more in line with the program's intentions and view points over the years. Hence, we may say that the conceptual validity of the observations and participatory testing is also growing over the years. For the future evaluation of the play-based curriculum this is an important finding. It seems advisable then, that a further improvement of mathematics education in the context of a play-based curriculum strategy should pay due attention to the teachers' style of observing and continuous monitoring in the classroom, as well as to the underlying (mathematical) epistemologies. For, especially in the education of younger children it is important that their introduction into mathematical culture is driven by a new, activity-based epistemology of the adults that opens possibilities for playfully dealing with mathematical ideas, instead of mechanically applying operations.

REFERENCES

- COBB, P., GRAVEMEIJER, K.P.E., YACKEL, E., MCKLAIN, K., & WHITENACK, J. (1997) Mathematizing and symbolising the emergence of chains of signification in one first-grade classroom, in: KIRSHNER, D. & WHITSON, J.A. (Eds.) *Situated Cognition. Social Semiotic, and Psychological Perspectives* (Erlbaum).
- CLARKE, D. (1996) Assessment, in: BISHOP, A.J., CLEMENTS, K. KEITEL, C., KILPATRICK, J., & LABORDE, C. *International Handbook of Mathematics Education* (327-370) (Kluwer).
- FENNEMA, E., & LOEF, M. (1992) Teachers' knowledge and its impact, in: GROUWS, D.A. (Ed.), *Handbook of Research on Learning and Teaching Mathematics* (147-164) (Macmillan).
- FIJMA, N., & VINK, H. (1998) *Op jou kan ik Rekenen* [I can count on you] (Van Gorcum).
- FUCHS, L. S., FUCHS, D., KARNS, K., HAMLETT, C.L., & KATZAROFF, M. (1999) Mathematics Performance Assessment in the Classroom: Effects on Teacher Planning and Student Problem Solving, *American Educational Research Journal*, Vol. 36, nr. 3, 609-646.
- GRAVEMEIJER, K. P.E. (1994) *Developing Realistic Mathematics Education* (CDβ-press).
- HOFER, B.K., & PINTRICHT, P.R. (1997) The development of epistemological theories: beliefs about knowledge and knowing and their relation to learning, *Review of Educational Research*, 67, 88-140.
- JANSSEN-VOS, F. (1997) *Basisontwikkeling* [Basic Development] (Van Gorcum).
- JANSSEN-VOS, F., POMPERT, B., & SCHIFERLI, T. (1998) *De HOREB Map* (APS).
- KNIJPSTRA, H., POMPERT, B., & SCHIFERLI, T. (1997) *Met jou kan ik Lezen en Schrijven* [With you I can read and write] (Van Gorcum).
- THOMPSON, A. (1992) Teachers' beliefs and conceptions. A synthesis of the research, in: GROUWS, D.A (Ed.) *Handbook of Research on Learning and Teaching Mathematics* (127-1460) (Macmillan).
- VAN OERS, B. (1994) Semiotic activity of young children in play: the construction and use of schematic representations, *European Early Childhood Education Research Journal*, 1994, Vol. 2, No. 1, 19-34.
- VAN OERS, B. (1996) Are you sure? The promotion of mathematical thinking in the play activities of young children, *European Early Childhood Education Research Journal*, 1996, Vol. 4, No. 1, 71-89.
- VAN OERS, B. (1999a) Education for the improvement of cultural participation, in: BROUGERE, G. & RAYNA, S. (Eds.) *Culture, Childhood, and Preschool Education /Culture, Enfance et Éducation Préscolaire* (pp 217-238) (Paris: UNESCO, 1999).
- VAN OERS, B. (1999b) Quality of diagnostic teaching abilities in early education, *European Early Childhood Education Research Journal*, Vol. 7, No. 2, 39-51.
- VERSCHAFFEL, L., GREER, B., & DE CORTE, E. (2000) *Making Sense of Word Problems* (Swets & Zeitlinger).
- WERTSCH, J.V. (1998) *Mind as Action* (Oxford University Press).

Correspondence about this paper should be addressed to:

Bert van Oers
Department of Education and Curriculum
Free University Amsterdam
Amsterdam
The Netherlands